Grade Level/Course: Algebra 1

Lesson/Unit Plan Name: Linear-Quadratic Systems

Rationale/Lesson Abstract: Students will be able to solve a Linear-Quadratic System algebraically and graphically.

Timeframe: 60 minutes

Common Core Standard(s): A-REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

<u>**Note:**</u> The Warm-Up is on page 9.

Solutions to the Warm-Up are on pages 10-11.

<u>**Additional Note:**</u> This lesson is the introductory lesson to Linear-Quadratic Systems. All of the Linear-Quadratic Systems in this lesson have integer ordered-pair solutions so that the systems can be solved by graphing without the use of technology.

A follow-up lesson could involve solving Linear-Quadratic Systems that do not have integer ordered-pair solutions. These systems can be solved algebraically using methods such as the quadratic formula or completing the square. Graphing these systems will not produce an exact solution and graphing technology can be used if available.

Instructional Resources/Materials: Graph paper, rulers

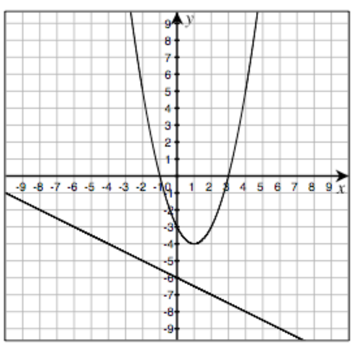
Activity/Lesson:

We have previously solved systems of linear equations both algebraically and graphically. We know that the solution to the system is the point of intersection of the two lines.

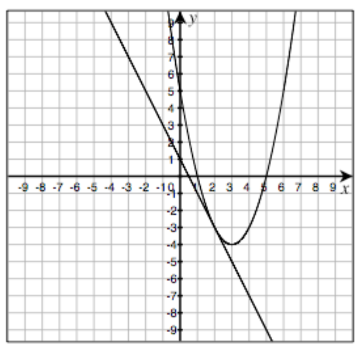
Now, we want to consider a system of equations where one function is linear and the other function is quadratic.

Turn to your elbow partner and discuss what the graph of a Linear-Quadratic System might look like. Will the graphs intersect? If so, how many times will they intersect? Have a few pairs share out their thoughts.

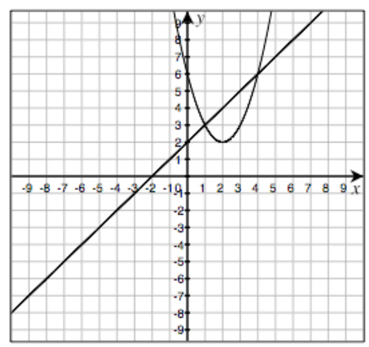
Let's look at the possibilities:



The line and the parabola do not intersect, so the linear-quadratic system has no solution.



The line and the parabola have one point of intersection, so the linear-quadratic system has one solution.



The line and the parabola have two points of intersection, so the linear-quadratic system has two solutions.

Example 1)

Solve the following system of equations both algebraically and graphically:

$f(x) = x^2 - 6x + 5$
$g(x) = x - 5$

Function notation is another way to represent the output values of the function. We know that $f(x) = y$ and $g(x) = y$. Now we can rewrite the system as follows:

$y = x^2 - 6x + 5$
$y = x - 5$

Notice that both the quadratic function and the linear function are equal to *y* so we can use the substitution method.

$$
\begin{aligned}
y &= x^2 - 6x + 5 \\
x - 5 &= x^2 - 6x + 5 \\
x - x - 5 &= x^2 - 6x - x + 5 \\
-5 &= x^2 - 7x + 5 \\
-5 + 5 &= x^2 - 7x + 5 + 5 \\
0 &= x^2 - 7x + 10 \\
0 &= (x - 2)(x - 5)
\end{aligned}
$$

Substitute $x - 5$ for y. The equation is quadratic so let's set equal to zero to solve the quadratic equation.

$x - 2 = 0$ or $x - 5 = 0$
$x = 2$ or $x = 5$

Since we have two *x* values, we know that this system has two solutions.

Now, we substitute the *x* values we found into either of the original functions to find the corresponding function value.
Choosing $g(x) = x - 5$ gives:

$g(x) = x - 5$	$g(x) = x - 5$
$g(2) = 2 - 5$	$g(5) = 5 - 5$
$g(2) = -3$	$g(5) = 0$

Notice that substituting into $f(x)$ results in the same outputs.

$f(x) = x^2 - 6x + 5$	$f(x) = x^2 - 6x + 5$
$f(2) = (2)^2 - 6(2) + 5$	$f(5) = (5)^2 - 6(5) + 5$
$f(2) = 4 - 12 + 5$	$f(5) = 25 - 30 + 5$
$f(2) = -8 + 5$	$f(5) = -5 + 5$
$f(2) = -3$	$f(5) = 0$

$\therefore$ the solutions to the system are the ordered pairs $(2,-3)$ and $(5,0)$.

Some students may prefer using the function $y = x - 5$ to evaluate $g(x) = x - 5$ when substituting for *x* to find the corresponding *y*.

How do the solutions to the system relate to the graphs of the two functions in the system?

Let's solve by graphing to find out!

$$f(x) = x^2 - 6x + 5$$
$$g(x) = x - 5$$

In looking at the structure of the quadratic function, we can see that it will factor. Factoring gives us an easy method for finding the *x*-intercepts.

$$f(x) = x^2 - 6x + 5$$
$$f(x) = (x-1)(x-5)$$
$$0 = (x-1)(x-5)$$

Set the function equal to zero to find the *x*-intercepts or zeros.

$$x - 1 = 0 \text{ or } x - 5 = 0$$
$$x = 1 \text{ or } x = 5$$

$\therefore$ the *x*-intercepts are the ordered pairs $(1,0)$ and $(5,0)$ so we can plot these points on our graph.

The *x*-coordinate of the vertex of the parabola will be the midpoint between $x = 1$ and $x = 5$ due to symmetry, so the vertex has an *x*-coordinate of $x = 3$.

Substituting $x = 3$ in to the function gives

$$f(x) = x^2 - 6x + 5$$
$$f(3) = (3)^2 - 6(3) + 5$$
$$f(3) = 9 - 18 + 5$$
$$f(3) = -9 + 5$$
$$f(3) = -4$$

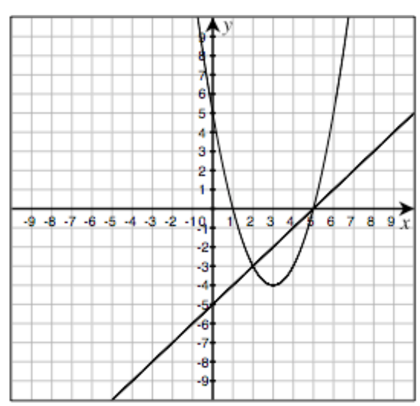
$\therefore$ the vertex has coordinates $(3,-4)$.

Plot other points on the parabola as necessary.

The other function in the system is $g(x) = x - 5$.

$g(x) = x - 5$ is a linear function with *y*-intercept $(0,-5)$ and slope or rate of change $= 1$.

By looking at the graph of the system, we can see that the graphs intersect at the points $(2,-3)$ and $(5,0)$ which are the solutions we found algebraically!



You-Try!

Solve the following system of equations both algebraically and graphically:

$f(x) = x^2 + 4x - 5$
$g(x) = 3x + 1$

<u>Answer to you-try:</u>

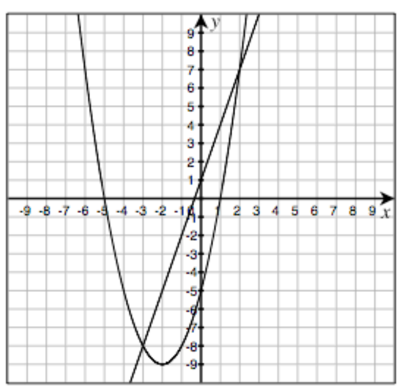
$$\begin{aligned} 3x + 1 &= x^2 + 4x - 5 \\ 0 &= x^2 + x - 6 \\ 0 &= (x-2)(x+3) \end{aligned}$$

$x - 2 = 0$ or $x + 3 = 0$
$x = 2$ or $x = -3$

$g(x) = 3x + 1$	$g(x) = 3x + 1$
$g(2) = 3(2) + 1$	$g(-3) = 3(-3) + 1$
$g(2) = 6 + 1$	$g(-3) = -9 + 1$
$g(2) = 7$	$g(-3) = -8$

$\therefore$ the solutions to the system are the ordered pairs $(2,7)$ and $(-3,-8)$.

Graph of the system:



The graphs intersect at the points $(2,7)$ and $(-3,-8)$ which are the solutions found algebraically.

Example 2)

Solve the following system of equations both algebraically and graphically:

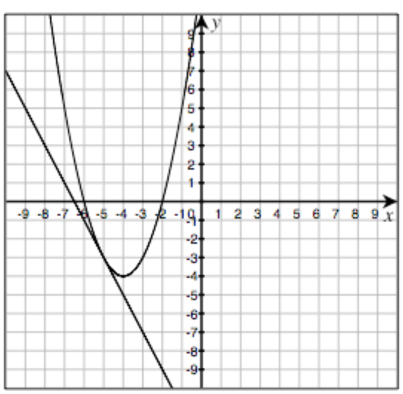
$$f(x) = -2x - 13$$
$$g(x) = x^2 + 8x + 12$$

$$\begin{aligned} y &= x^2 + 8x + 12 \\ -2x - 13 &= x^2 + 8x + 12 \\ 0 &= x^2 + 10x + 25 \\ 0 &= (x+5)^2 \\ 0 &= x + 5 \\ x &= -5 \end{aligned}$$

$$\begin{aligned} f(x) &= -2x - 13 \\ f(-5) &= -2(-5) - 13 \\ f(-5) &= 10 - 13 \\ f(-5) &= -3 \end{aligned}$$

$\therefore$ this system has only one solution and the solution is the ordered pair $(-5,-3)$.

Graph of the system:



Refer to Example 1 for a method for graphing a quadratic function that is factorable, or take the opportunity to review other methods for graphing quadratic functions for cases when the quadratic function does not factor.

We can see that the line only intersects the parabola once at the point $(-5,-3)$.

Partner Work:

Given the following system:

$$f(x) = 3x + 7$$
$$g(x) = -x^2 - 2x + 3$$

Partner A: Solve the system algebraically.

Partner B: Solve the system graphically.

Both partners: Write one sentence describing the relationship between the graph of the system and the solution to the system.

When each partner is finished, students share their work and make sure they both found the same solution to the system. Also, students trade sentences and compare what they each wrote.

Answers to partner work:

Partner A:

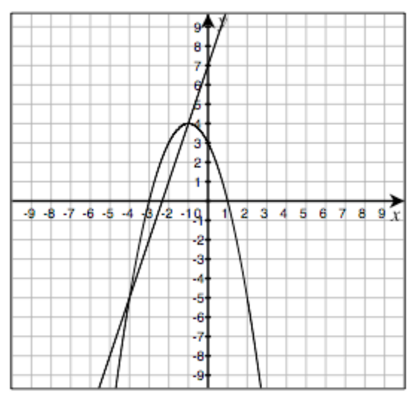
$$3x + 7 = -x^2 - 2x + 3$$
$$x^2 + 5x + 4 = 0$$
$$(x+1)(x+4) = 0$$

$$x + 1 = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = -1 \quad \text{or} \quad x = -4$$

$$f(x) = 3x + 7 \qquad f(x) = 3x + 7$$
$$f(-1) = 3(-1) + 7 \qquad f(-4) = 3(-4) + 7$$
$$f(-1) = -3 + 7 \qquad f(-4) = -12 + 7$$
$$f(-1) = 4 \qquad f(-4) = -5$$

Partner B:

Graph of the system:



$\therefore$ the solutions to the system are the ordered pairs $(-1,4)$ and $(-4,-5)$.

The sentence they write should say that the solution to the system is the point or points of intersection of the graphs.

Exit Ticket:

Which of the statements are true about the system of equations below?

$f(x) = x^2 + 4x + 5$
$g(x) = -2x - 3$

A) The system consists of two lines. ○ True ○ False

B) The system consists of a line and a parabola. ○ True ○ False

C) The system has only one solution. ○ True ○ False

D) The system has two solutions. ○ True ○ False

E) $(-4,5)$ is a solution to the system. ○ True ○ False

Answers to Exit Ticket:

A) False
B) True
C) False
D) True
E) True

Warm-Up

Review: CA Alg. 1 CCSS A-REI.4b

Which equations can be used to find a root of $3x^2 + 7x + 2 = 0$ when factoring and using the zero product property?

A) $x + 1 = 0$

B) $x + 2 = 0$

C) $3x + 2 = 0$

D) $3x + 7 = 0$

E) $3x + 1 = 0$

Review: CA Alg. 1 CCSS A-REI.6

Solve the system of linear equations using two different methods. Be sure to name each method you use.

$y = x + 3$

$y = 2x + 4$

Current: CA Alg. 1 CCSS F-IF.4

Which of the following statements are true about the graph of $f(x) = (x-3)^2$?

A) The graph is the same shape as $y = x^2$.

B) The graph is more narrow than $y = x^2$.

C) The vertex is a minimum.

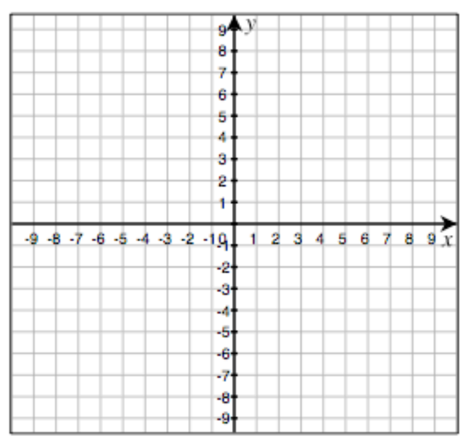
D) The vertex is $(3,0)$.

E) The vertex is $(0,3)$.

F) The range of $f(x)$ is $f(x) \ge 0$.

Current: CA Alg. 1 CCSS F-IF.7a

Graph the function $f(x) = -(x-3)^2$ on the axes provided below. Plot at least five key points on the graph.



Write down similarities and differences between the graph of this function, and the graph of $f(x) = (x-3)^2$.

Answers to warm-up:

Quadrant I

Substitution Method:

$$y = x + 3$$
$$y = 2x + 4$$

$$2x + 4 = x + 3$$
$$2x - x + 4 = x - x + 3$$
$$x + 4 = 3$$
$$x + 4 - 4 = 3 - 4$$
$$x = -1$$

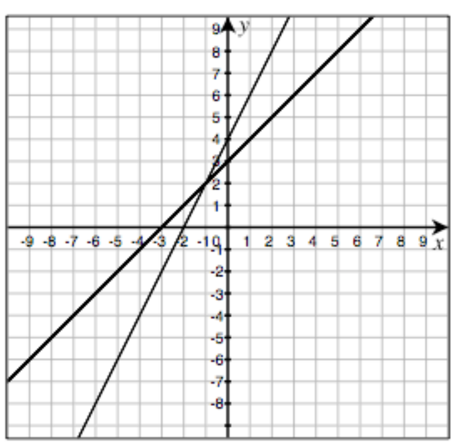
Using the equation $y = 2x + 4$:

$$y = 2x + 4$$
$$y = 2(-1) + 4$$
$$y = -2 + 4$$
$$y = 2$$

$\therefore$ the solution to the system is the ordered pair $(-1,2)$.

Graphical Method:

Graph of the system:



$\therefore$ the solution to the system is the ordered pair $(-1,2)$.

Quadrant II

Which equations can be used to find a root of $3x^2 + 7x + 2 = 0$ when factoring and using the zero product property?

$$3x^2 + 7x + 2 = 0$$
$$(3x + 1)(x + 2) = 0$$

The correct answer choices are: B and E

Quadrant III

Which of the following statements are true about the graph of $f(x) = (x - 3)^2$?

The function is a horizontal translation, 3 units to the right, of the graph of $f(x) = x^2$.

There is no vertical stretch or horizontal compression so the shape of the graph of $f(x) = (x - 3)^2$ is the same as the shape of $f(x) = x^2$.

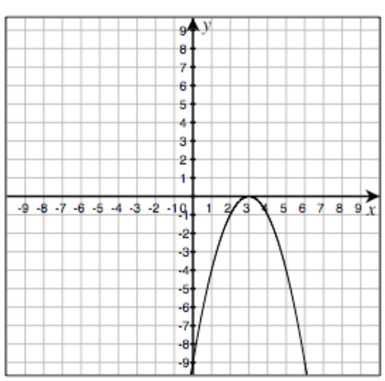
The vertex of $f(x) = (x - 3)^2$ is $(3,0)$, and since the function is concave up, the vertex is a minimum.

The range of the function is $f(x) \geq 0$.

The correct answer choices are: A, C, D and F

Quadrant IV

Graph the function $f(x) = -(x - 3)^2$ on the axes provided below. Plot at least five key points on the graph.



Write down similarities and differences between the graph of this function, and the graph of $f(x) = (x - 3)^2$.

The graphs have the same vertex and the same shape.

$f(x) = (x - 3)^2$ is concave up $\therefore$ the vertex is a minimum.
$f(x) = -(x - 3)^2$ is concave down $\therefore$ the vertex is a maximum.

The range of $f(x) = (x - 3)^2$ is $f(x) \geq 0$
The range of $f(x) = -(x - 3)^2$ is $f(x) \leq 0$.